String MSSM through flipped SU(5) from $\mathbf{Z}_{12}$ orbifold

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In a Z_{12-I} orbifold compactification through an intermediate flipped SU(5), the string MSSM ($\mathcal{S}$MSSM) spectra (three families, one pair of Higgs doublets, and neutral singlets) are obtained with the Yukawa coupling structure. The GUT $\sin^2\theta_W^0=\frac{3}{8}$, even with exotics in the twisted sector, can be run to the observed electroweak scale value by mass parameters of vectorlike exotics near the GUT scale. We also obtain R-parity and doublet-triplet splitting.

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Superstring theory is with us for more than 20 years. Yet there has not appeared any unique low energy string prediction. Some relevant phenomenological and cosmological issues are $b-\tau$ unification [1], unification of gauge couplings at $M_{\rm GUT}$ [2], the GUT value of $\sin^2\theta_W$ [3], and existence of fractionally charged particles (FCP) [4]. Even though these seem to be most esthetic, any of these is not an inevitable prediction of string theory. But, if string theory is valid as a particle physics model, one firm prediction is that it must lead to the standard model (SM) or the minimal supersymmetric standard model (MSSM), with some inclusion of SM singlets.

Obtaining MSSM through simple group GUTs, such as SU(5), SO(10) and E$_6$, is attractive since it can explain the first three issues without any FCPs. This simple group GUT scheme needs adjoint representations for the GUT scale spontaneous symmetry breaking. However, obtaining adjoint representations from superstring theory is very difficult if not impossible [5, 6, 7]. So, it has been a recent trend [8] in string compactification to consider non-simple group GUTs such as flipped SU(5) [9], trinification, and Pati-Salam model. Among these, let us focus on flipped SU(5).

As pointed out recently [10], flipped SU(5) has many nice features, among which the possibility of GUT symmetry breaking by a rank-lowering (rank 5→ rank 4) scalar field is most relevant in string phenomenology. Representation **10** achieves this symmetry breaking pattern. This view was taken in the fermionic construction by Antoniadis, Ellis, Hagelin, and Nanopoulos (NAHE) [11]. With the NAHE set of conditions in the fermionic scheme, there appear $Q_{\rm em}=\pm\frac12$ particles transforming as $\mathbf{4}$ or $\overline{\mathbf{4}}$ under a hidden SU(4)$'$, but it has been known that the resulting cryptons (group-singlet composite states under the confining gauge group) are integer-charged [4]. On the cosmological side, cryptons with mass in the range $10^{11}-10^{13}$ GeV have been suggested to present an important signal via their decay products now in the universe [12]. Therefore, it is an important phenomenological issue whether string derived flipped SU(5) models always predict integer-charged cyptons. Our orbifold construction of flipped SU(5) show that cryptons are integer-charged if cryptons are indeed formed. In one construction, we obtain SU(4)$'$ [13], but with the hidden sector gauge group and the spectrum different from those of [4]. In this letter, we consider another construction where the nonabelian part of hidden sector is SU(2)$'\times$SO(10)$'$. This is the first example realizing a flipped SU(5) without SU(4)$'$. Most probably, the compactification allows color exotics (C-exotics), the colored particles not having $Q_{\rm em}=\frac23(\frac{-2}{3})$ or $\frac{-1}{3}(\frac13)$ for color (anti-)triplets, and similarly defined electromagnetic exotics (E-exotics) and flipped SU(5) GUT exotics (G-exotics). In the model, there appear G-exotics (including $Q_{\rm em}=\pm\frac16$ C-exotics in them) and $Q_{\rm em}=\pm\frac12$ E-exotics. If lucky, discovery of $Q_{\rm em}=\pm\frac12$ exotics at low energy can be a signal to string origin of elementary particles.

For **10** and $\overline{\mathbf{5}}$ representations of SU(5) GUT and neutral singlet ν^c, one exchanges $u^c\leftrightarrow d^c$ and $e^c\leftrightarrow\nu^c$ to obtain flipped SU(5). Thus, the matter representation of flipped SU(5) is represented under SU(5)$\times$U(1)$_{\rm X}$ as

$$\begin{aligned}&\mathbf{16}_{\rm flip}\equiv\mathbf{10_1}+\overline{\mathbf{5}}_{\mathbf{-3}}+\mathbf{1_5}=(d^c,q,\nu^c)+(u^c,l)+e^c,\\&\mathbf{5_{-2}}=(D,h_d),\ \overline{\mathbf{5}}_{\mathbf{2}}=(\overline{D},h_u)\end{aligned}\tag{1}$$

where q and l are lepton and quark doublets, D is $Q_{\rm em}=-\frac13$, and $h_{d,u}$ are Higgs doublets giving mass to d,u quarks. Note that $\mathbf{10_1}$, which does not have a weight at the center of the weight diagram, contains the neutral component ν^c; thus a VEV of $\mathbf{10_1}$ lowers the rank and breaks flipped SU(5) down to the SM group. The electroweak hypercharge of (1) is given by $Y=\frac15(X+Y_5)$, where $Y_5={\rm diag.}(\frac13\ \frac13\ \frac13\ \frac{-1}{2}\ \frac{-1}{2})$, and $X={\rm diag.}(x\ x\ x\ x\ x)$. Spontaneous symmetry breaking of flipped SU(5) proceeds in two steps via VEVs of $(\mathbf{10_1}+\overline{\mathbf{10}}_{\mathbf{-1}})$ at the GUT scale and $(\mathbf{5_{-2}}+\overline{\mathbf{5}}_{\mathbf{2}})$ at the the electroweak scale. In addition to three $\mathbf{16}_{\rm flip}$s, thus

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flipped SU(5) spectra must include $\mathbf{10_1}, \overline{\mathbf{10}}_{\mathbf{-1}}, \mathbf{5_{-2}}$, and $\overline{\mathbf{5}}_{\mathbf{2}}$. These Higgs multiplets needed for spontaneous symmetry breaking must be vector-like.

The appearance of fractionally charged particles is a generic phenomenon in string models, because electromagnetic charge $Q_{\rm em}$ is not necessarily embedded in an SU(5)-like form. In addition, in most standard-like models, the unification value $\sin^2\theta_W^0$ turns out to be $\le \frac{3}{8}$.

String MSSM ($\mathcal{S}$MSSM) is defined to be *string compactification with spectra compatible with obtaining* MSSM. It is not a 'string inspired' MSSM but 'string derived' MSSM. The particle contents of $\mathcal{S}$MSSM are three families of quarks and leptons, one pair of Higgs doublets without colored scalars, and GUT scale Higgs bosons responsible for breaking a GUT group down to the SM. Models with no possibility of fitting $\sin^2\theta_W \simeq 0.23$ at the electroweak scale are excluded from $\mathcal{S}$MSSM. But string derived standard-like models with $\sin^2\theta_W \simeq 0.23$ at the electroweak scale are included in $\mathcal{S}$MSSM. $\mathcal{S}$MSSMs should possess desirable Yukawa couplings, not obviously excluded by phenomenology. Earlier standard-like models [14] are not $\mathcal{S}$MSSMs.

The orbifold compactification [15] got much interest due to its geometric nature and its simplicity in model building. To construct flipped SU(5) in the orbifold scheme, we adopt $\mathbf{Z}_{12-I}$ twist. If one does not want to introduce any nonabelian group except SU(5) from the E$_8$ part by one shift V, $\mathbf{Z}_{12-I}$ and $\mathbf{Z}_{12-II}$ are the only possible twists, which can be easily checked from the Dynkin diagram technique [16], as discussed in the expanded version of this letter [13]. The $\mathbf{Z}_{12-I}$ shift in the six real (or three complex) internal space is taken as [7],

$$\phi_s = (\tfrac{5}{12},\ \tfrac{4}{12},\ \tfrac{1}{12}) \quad \text{with} \quad \phi_s^2 = \tfrac{1}{12}\cdot\tfrac{7}{2}. \tag{2}$$

This $\mathbf{Z}_{12-I}$ shift is of order 12 for the first and third tori and is order 3 for the second torus. Internal gauge fields can wind torus, which are called Wilson lines (WLs).

For the shift and Wilson lines (WLs), we take the following,

$$\begin{aligned} V &= \left(\tfrac14\ \tfrac14\ \tfrac14\ \tfrac14\ \tfrac14\ \tfrac{5}{12}\ \tfrac{6}{12}\ 0\right)\left(\tfrac{2}{12}\ \tfrac{2}{12}\ 0\ 0^5\right)\\ a_3 &= a_4 = \left(0^5\ 0\ \tfrac{-1}{3}\ \tfrac13\right)\left(0\ 0\ \tfrac23\ 0^5\right)\end{aligned} \tag{3}$$

which satisfy all the conditions and give $V^2-\phi_s^2=\frac12$ [7, 15]. In twisted sectors wound by WLs, we must consider $V_+ \equiv V + a_3$ and $V_- \equiv V - a_3$. For twisted sectors not wound by WLs, we use $V_0 \equiv V$. In twisted sectors wound by WLs, $V_+^2-\phi_s^2=\frac56$ and $V_-^2-\phi_s^2=\frac32$ are used in the multiplicity ($\mathcal{P}$) calculation.

In the low energy world, massless modes are important. Massless modes appear in the untwisted sector U (like bulk modes in extra dimensional field theory) and in the twisted sectors T_f (like localized modes at fixed points f in extra dimensional field theory). We are interested in obtaining massless fields of the $\mathrm{E}_8\times\mathrm{E}_8'$ gauge sector of heterotic string. The masslessness conditions for left and right movers must be satisfied simultaneously, $M_L^2 = M_R^2 = 0$.

Our discussion will proceed in two steps: firstly find all massless modes possessing $\mathcal{N}=1$ supersymmetry and second derive Yukawa couplings. Supersymmetry in heterotic string is the symmetry of the NS and R sectors of right movers. The R sector is represented as four component half integers $s\equiv(s_0,\tilde s)$. The allowed values of s determine the chirality (by s_0 component) and U_i (by $\tilde s$) of the untwisted sector fields. The supersymmetry condition for twist (2) is (See Eq. (3.58) of Ref. [7]), $\tilde r = (-\frac12,\frac12,\frac12)$ so that $\phi_s\cdot\tilde r = 0$.

Untwisted sector U: The masslessness condition in U is given by $P^2=2$. Then, always one can find a massless condition for right movers. We define $U_i \equiv \tilde s + \tilde r$ for untwisted sector field nontrivial at the ith torus as $U_1=(-1,0,0)$, $U_2=(0,1,0)$, $U_3=(0,0,1)$, which depends on what $\tilde s$ is. The above U_i correspond to the untwisted $\tilde s = (---), (++-), (+-+)$, respectively.

a. Gauge group : The gauge multiplet is found with left movers by winding momenta $P_{\rm un}$ of length $P_{\rm un}^2=2$, satisfying $P_{\rm un}\cdot V=0$ and $P_{\rm un}\cdot a_3=0$. So, we find the unbroken gauge group as

$$[SU(5)\times U(1)_X\times U(1)^3]\times[SU(2)\times SO(10)\times U(1)^2]'. \tag{4}$$

When flipped SU(5) is broken, three U(1)s spanning over the SU(5) entry region, satisfying $P_{\rm un}\cdot Q_i=0$, are considered: $Q_1 = (1\ 1\ 1\ 0\ 0\ 0\ 0\ 0)()', Q_2 = (0\ 0\ 0\ 1\ 1\ 0\ 0\ 0)()', Q_X = (-2\ -2\ -2\ -2\ -2\ 0\ 0\ 0)()'$. In flipped SU(5) with the above Q_X charge, we obtain $\sin^2\theta_W=\frac38$, from Eq. (10.28) of [7], $\sin^2\theta_W = 1/(1+\sum_i c_i^2)$, where c_i are properly defined normalization of U(1)s.

b. Matter in U : For $P_{\rm un}\cdot V = k/12$ where $P_{\rm un}^2=2$ and $k=1,\cdots,11$, also there can appear massless states. They are interpreted as matter. The $\mathcal{CPT}$ conjugate of k-bin spectrum appears in $(12-k)$-bin. Thus, in $k=6$ the $\mathcal{CPT}$ conjugates appear again in $k=6$.

Twisted sectors T_k: The $k^{\rm th}$ twisted sector is distinguished by $kV, k(V+a_3)$, and $k(V-a_3)$, which are denoted as $\tilde V_{0,+,-}$. For right movers, the twist in the $k^{\rm th}$ twisted sector is $\tilde\phi\equiv k\phi_s$. The masslessness conditions for left and right movers are

$$(P+\tilde V)^2 = 2(1-c_k)^L - 2\tilde N_L,\ \ (s+\tilde\phi)^2 = 2(1-c_k)^R \tag{5}$$

where $\tilde N_L$ is the left oscillator number in the $k^{\rm th}$ twisted sector, and $2(1-c_k)^L = \frac{210}{144}(k=1), \frac{216}{144}(k=2), \frac{234}{144}(k=3), \frac{192}{144}(k=4), \frac{210}{144}(k=5), \frac{216}{144}(k=6)$ for the left movers, and $2(1-c_k)^R = \frac{11}{24}(k=1), \frac12(k=2), \frac58(k=3), \frac13(k=4), \frac{11}{24}(k=5), \frac12(k=6)$ for the right movers. It is sufficient to consider $k=1,2,\cdots,6$ twisted sectors only.

The multiplicity $\mathcal{P}$ satisfying the orbifold condition in the $m^{\rm th}$ twisted sector is given by one loop partition function of string [7, 15],

$$\mathcal{P}_m = \frac{1}{N}\sum_{k=0}^{N-1} \chi(\theta^m, \theta^k)\ e^{2\pi i k \Theta_0}\ , \tag{6}$$

where $N = 12$ for $\mathbf{Z}_{12}$ orbifolds, and

$$\begin{aligned}\Theta_0 = &\sum_j (N^L_j - N^R_j)\hat{\phi}_j - \frac{m}{2}(V_0^2 - \phi_s^2) \\ &+ (P + mV_0)\cdot V_0 - (\tilde{s} + m\phi_s)\cdot\phi_s + \text{ integer}\end{aligned} \tag{7}$$

where j denotes the coordinates of the 6 dimensional compactified space running over $\{1, 2, 3, \bar{1}, \bar{2}, \bar{3}\}$ in complexified coordinates, and $\hat{\phi}_j = \phi_{sj}\mathrm{sgn}(\tilde{\phi}_j)$ where $\mathrm{sgn}(\tilde{\phi}_{\bar{j}}) = -\mathrm{sgn}(\tilde{\phi}_j)$. Here $\tilde{\phi}_i \equiv k\phi_i$ mod. Z such that $0 < \tilde{\phi}_i \le 0$, $\tilde{\phi}_{\bar{i}} \equiv -k\phi_i$ mod. Z such that $0 < \tilde{\phi}_{\bar{i}} \le 0$, and $\phi_{s\bar{j}} \equiv \phi_{sj}$. (If $k\phi_i$ is an integer, $\tilde{\phi}_j = 1$ [18, 19]). The $\chi(\theta^m, \theta^k)$ in Eq. (6) denotes the degenerate factor tabulated in Appendix D of [7]. For the sectors wound by WLs, $V_\pm$ are used instead of V_0. The untwisted sector $k = 0$ and twisted sectors for $k = 3, 6, 9$ are not affected by WLs since the WL condition, $3a_3 = 0$, makes it trivial. So, for $k = 3, 6, 9$, there is the additional condition, $(P + kV)\cdot a_3 = 0$. For $k \ne 3, 6, 9$, the multiplicity for each twisted sector $k(V + m_k a_3)$ is $\mathcal{P} = \frac{1}{3}\mathcal{P}_k$.

Now it is straightforward to calculate the massless modes in each sector. For flipped SU(5) fields, the result is summarized in Table I. In U only, we included the group singlet $\mathbf{1_0}$. There does not appear any massless matter field transforming nontrivially under SO(10)$'$. In T_1 and T_5, there appear G-exotics. We have not listed 20 SU(2)$'$ doublets, 14 E-exotics with $Q_{\rm em}=\pm\frac12$ and 79 $Q_{\rm em}=0$ singlets. Five extra U(1)s are also shown, which can be broken at high energy scales.

Yukawa couplings: Rules for Yukawa couplings are summarized in [7, 20]. For the coupling $U_1^k U_2^l U_3^m T^p$ where $T^p \sim T_{k_1}^{n_1} T_{k_2}^{n_2}\cdots$ with $p = n_1k_1 + n_2k_2 + \cdots$, the Lorentz invariance rule for the $\mathbf{Z}_{12-I}$ shift is $(-k + \frac{5}{12}p_1, l + \frac{4}{12}p_2, m + \frac{1}{12}p_3) = (-1, 1, 1)\mathrm{mod.}(n_1, n_2, n_3)$ where n_1, n_2, and n_3 for $\mathbf{Z}_{12-I}$ is $(12, 3, 12)$, and p_1, p_2 and p_3 are calculated using the H-momenta. The invariance under a generalized Lorentz shift $k\phi_s$ in T_k gives the H-momentum conservation. The H-momenta for $\mathbf{Z}_{12-I}$ twist are

$$\begin{aligned}&T_1 : (\tfrac{-7}{12}\ \tfrac{4}{12}\ \tfrac{1}{12}),\quad T_2 : (\tfrac{-1}{6}\ \tfrac{4}{6}\ \tfrac{1}{6}),\quad T_3 : (\tfrac{1}{4}\ 0\ \tfrac{-3}{4}) \\ &T_4 : (\tfrac{-1}{3}\ \tfrac{1}{3}\ \tfrac{1}{3}),\quad T_5 : (\tfrac{1}{12}\ \tfrac{-4}{12}\ \tfrac{-7}{12}),\quad T_6 : (\tfrac{-1}{2}\ 0\ \tfrac{1}{2}).\end{aligned} \tag{8}$$

Consider T_6, for example. If we consider T_6^2, we have $(-1, 0, 1)$, thus we supply $(0, 1, 0)$ by U_2, and hence $T_6^2 U_2$ is allowed. For the untwisted sector fields only, there is no U and UU coupling. But cubic couplings can be present in the form $U_1U_2U_3$. There is no quadratic coupling. All the allowed cubic terms are, using T_7 instead of T_5,

$$U_1U_2U_3, T_6T_6U_2, T_4T_4T_4, T_2T_4T_6, T_1T_4T_7. \tag{9}$$

Sct.	$P\cdot V;\tilde{s}\to U_i$	$(\mathbf{SU(5)})^\chi_{(\mathbf{U(1)_X};U(1)^3;U(1)'^2)}$	$\mathcal{P}$
	$\frac{1}{12};(+-+)\to U_3$	$\overline{\mathbf{10}}^L_{-\mathbf{1};0^5},\ \mathbf{5}^L_{\mathbf{3};0^5},\ \mathbf{1}^L_{-\mathbf{5};0^5}$	1
U	$\frac{4}{12};(++-)\to U_2$	$\overline{\mathbf{5}}^L_{\mathbf{2};0^5}$	1
	$\frac{5}{12};(+++)\to U_1$	$\mathbf{10}^R_{\mathbf{1};0^5},\ \overline{\mathbf{5}}^R_{-\mathbf{3};0^5},\ \mathbf{1}^R_{\mathbf{5};0^5}$	1
	$\frac{4}{12};(++-)\to U_2$	$\mathbf{1}^L_{\mathbf{0};0^5}$	1

Sct.	$\tilde{s}\to\chi$	$(\mathbf{SU(5)})^\chi_{(\mathbf{U(1)_X};U(1)^3;U(1)'^2)}$	$\mathcal{P}$
		$\overline{\mathbf{10}}^L_{-\mathbf{1};\frac12,0,0;0,0}$	4
T_6	$(-\pm-)\to R,L$	$\mathbf{10}^L_{\mathbf{1};\frac{-1}{2},0,0;0,0}$	3
		$\mathbf{5}^L_{\mathbf{3};\frac12,0,0;0,0},\ \overline{\mathbf{5}}^L_{-\mathbf{3};\frac{-1}{2},0,0;0,0}$	2
		$\mathbf{1}^L_{\mathbf{5};\frac{-1}{2},0,0;0,0},\mathbf{1}^L_{-\mathbf{5};\frac12,0,0;0,0}$	2
T_1^0		$\overline{\mathbf{5}}^L_{\frac{-\mathbf{1}}{\mathbf{2}};\frac{-7}{12},\frac{6}{12},0;\frac16,\frac16},\ \overline{\mathbf{5}}^L_{\frac{-\mathbf{1}}{\mathbf{2}};\frac{5}{12},\frac{-6}{12},0;\frac16,\frac16}$	1
T_1^0	$(---)\to L$	$\mathbf{5}^L_{\frac{\mathbf{1}}{\mathbf{2}};\frac{-1}{12},0,\frac{6}{12};\frac16,\frac16}$	1
T_1^-		$\mathbf{5}^L_{\frac{\mathbf{1}}{\mathbf{2}};\frac{-1}{12},\frac{4}{12},\frac{2}{12};\frac16,\frac16}$	1
T_2^0	$(---)\to L$	$\mathbf{5}^L_{\mathbf{3};\frac{-1}{6},0,0;0,0},\ \mathbf{1}^L_{-\mathbf{5};\frac{-1}{6},0,0;0,0}$	1
T_4^0	$(---)\to L$	$\mathbf{5}^L_{-\mathbf{2};\frac{-1}{3},0,0;0,0}$	3
		$\overline{\mathbf{5}}^L_{\mathbf{2};\frac{-1}{3},0,0;0,0}$	2
T_5^0		$\mathbf{5}^R_{\frac{\mathbf{1}}{\mathbf{2}};\frac{7}{12},0,\frac{6}{12};\frac{-1}{6},\frac{-1}{6}},\ \mathbf{5}^R_{\frac{\mathbf{1}}{\mathbf{2}};\frac{-5}{12},0,\frac{-6}{12};\frac{-1}{6},\frac{-1}{6}}$	1
T_5^0	$(-+-)\to R$	$\overline{\mathbf{5}}^R_{-\frac{\mathbf{1}}{\mathbf{2}};\frac{1}{12},\frac{6}{12},0;\frac{-1}{6},\frac{-1}{6}}$	1
T_5^-		$\overline{\mathbf{5}}^R_{\frac{-\mathbf{1}}{\mathbf{2}};\frac{1}{12},\frac{2}{12},\frac{4}{12};\frac{-1}{6},\frac{-1}{6}}$	1

TABLE I: A $\mathbf{Z}_{12-I}$ orbifold spectra of the flipped SU(5) sector. These are SU(2)$'$ × SO(10)$'$ singlets. In T_6, the $\mathcal{CPT}$ conjugates appear again in T_6.

We tabulated some H-quantum numbers for singlet combinations which enable us to search for higher order terms [13]. Thus, we obtain the following.

(i) *MSSM spectrum*: G-exotics $\mathbf{5}_{\frac{\mathbf{1}}{\mathbf{2}}}$ and $\overline{\mathbf{5}}_{-\frac{\mathbf{1}}{\mathbf{2}}}$ appear in T_1 and T_5. They form vector-like representations and can be removed by $T_1T_4T_7$ couplings with VEVs of singlets in T_4. In T_4, there are 42 $Q_{\rm em}$=0 singlets, which make the removal possible. E-exotics with $Q_{\rm em}=\pm\frac12$ appear in T_1 and T_5, which can be removed again by $T_1T_4T_7$ couplings.

There exist $\{\mathbf{10_1}, \overline{\mathbf{10}}_{-\mathbf{1}}\}$ whose VEV ($\langle\nu^c\rangle$) breaks the flipped SU(5) down to the SM. Also, there exist the needed electroweak Higgs fields $h_{d,u} \in \{\mathbf{5_{-2}}, \overline{\mathbf{5}}_{\mathbf{2}}\}$.

In T_6 and T_4, there appear vectorlike representations $(\mathbf{10_1} + \overline{\mathbf{10}}_{-\mathbf{1}})$s, $(\mathbf{1_5} + \mathbf{1_{-5}})$s, and $(\mathbf{5_{-2}} + \overline{\mathbf{5}}_{\mathbf{2}})$s. The vectorlike representations in T_6 are removed by T_6T_6 times singlet couplings [13]. The vectorlike representation in T_4, $2(\mathbf{5_{-2}} + \overline{\mathbf{5}}_{\mathbf{2}})^L$, can be removed by $T_4T_4T_4$ couplings where $\mathbf{1_0}$ in T_4 gets a VEV. Thus, $2(\mathbf{1_5} + \mathbf{1_{-5}})^R, 2(\overline{\mathbf{5}}_{-\mathbf{3}} + \mathbf{5_3})^R, 3(\mathbf{10_1} + \overline{\mathbf{10}}_{-\mathbf{1}})^R$, and $2(\mathbf{5_{-2}} + \overline{\mathbf{5}}_{\mathbf{2}})^L$ are removed at the GUT scale. In all these, several singlets with $Q_X = 0$ are expected to develop GUT scale VEVs. Then, we obtain $\mathbf{16}^R_{\rm flip}(U_1) + \mathbf{16}^R_{\rm flip}(U_3) + \mathbf{5}^R_{-\mathbf{2}}(U_2)$ from untwisted sectors, and $\mathbf{10}^R_{\mathbf{1}}(T_6), \mathbf{1}^R_{\mathbf{5}}(T_2), \overline{\mathbf{5}}^R_{-\mathbf{3}}(T_2), \overline{\mathbf{5}}^R_{\mathbf{2}}(T_4)$ from twisted sectors. These constitute three families and one pair of Higgs quintets. It is interesting to note that the pair of

Higgs quintets, $\mathbf{5_{-2}}(U_2)$ [21], and $\overline{\mathbf{5}}_\mathbf{2}(T_4)$, survives the GUT scale symmetry breaking. Certainly, it is not allowed to write $M_{\rm GUT}\mathbf{5_{-2}}(U_2)\overline{\mathbf{5}}_\mathbf{2}(T_4)$ since there is no coupling of the form U_2T_4. Also, $S_0U_2T_4$ with some singlet S_0 is not allowed since (9) does not include such a term. Thus, the coupling $\mathbf{5_{-2}}(U_2)\overline{\mathbf{5}}_\mathbf{2}(T_2)$ must arise from higher order terms, suppressing the Higgs doublet mass far below the GUT scale. But there exists the coupling of the type $U_2T_6T_6$ where $U_2 = \mathbf{5_{-2}}$ and $T_6 = \mathbf{10_1}$ among $2(\mathbf{10_1} + \overline{\mathbf{10}}_\mathbf{-1})$ in T_6. We require that $\langle\mathbf{10_1}\rangle = \langle\overline{\mathbf{10}}_\mathbf{-1}\rangle \sim M_{\rm GUT}$ by $\langle\nu^c\rangle$. It was shown that this coupling is crucial in realizing the doublet triplet splitting in flipped SU(5) [10, 11, 22]. We have all the types of fields needed for the doublet-triplet splitting discussed in [22]. Thus considering cubic couplings, we obtain the so-called MSSM spectra with one pair of Higgs doublets. However, the survival hypothesis [23] is applicable here also if we include all the higher order terms. Indeed, there exist higher order terms for $\mathbf{5_{-2}}(U_2)\overline{\mathbf{5}}_\mathbf{2}(T_2)$, which however can be made sufficiently small [13].

(ii) *R-parity*: If we consider cubic couplings of (9), we can define an R-parity in the standard way, $R = -1$ for matter fermions and $R = +1$ for Higgs bosons. A nontrivial parity can be defined as $R = -1$ for $\mathbf{10_1}(U_1)$, $\mathbf{10_1}(U_3)$, $\overline{\mathbf{5}}_\mathbf{-3}(U_1)$, $\overline{\mathbf{5}}_\mathbf{-3}(U_3)$, $\mathbf{1_5}(U_1)$, $\mathbf{1_5}(U_3)$ and $R = +1$ for $\mathbf{5_{-2}}(U_2)$. Mixing between the first two families in the untwisted sector and the third family in T_5 and T_2^0 is always possible if VEVs of some neutral singlets are supposed. Such neutral singlets should preserve all symmetries relevant at low energies. Even with the mixing terms between untwisted and twisted matter fields, the R-parity relevant in low energies can still be defined by assigning $R = 1$ for the neutral singlets developing VEVs, and $R = -1$ for $\mathbf{10_1}(T_6)$, $\overline{\mathbf{5}}_\mathbf{-3}(T_2)$, $\mathbf{1_5}(T_2)$. Then, the allowed Yukawa coupling $T_6T_2T_4$ determines $R = +1$ for $\overline{\mathbf{5}}_\mathbf{2}(T_4)$. Thus, R-parity can survive down to low energies and hence R-parity conservation for proton longevity is fulfilled in the present model.

(iii) *Quark mixing*: For mixing of fermions, we choose the quark mixing, since there is one coupling relevant only for one quark, $T_6T_2T_4$, which is interpreted as the top quark Yukawa coupling, $\mathbf{10_1}\overline{\mathbf{5}}_\mathbf{-3}\overline{\mathbf{5}}_\mathbf{2}$. Then, b quark mass arises in terms of U_2 Higgs doublet through $T_6T_6U_2$. But τ may be placed in the untwisted sector because the charged lepton in twisted sector obtain mass at higher order [13]. If the coupling strength of $T_6T_2T_4$ and $T_6T_6U_2$ are comparable, a large $\tan\beta$ is needed to obtain $m_t/m_b \sim 35$. So, two light quark families are placed in the untwisted sector. These have the cubic couplings of the form $U_1U_2U_3$, rendering $Q_{\rm em} = -\frac{1}{3}$ quarks mass. For $m_b \gg m_s$, it is assumed that this U^3 coupling strength is much smaller than that of $T_6T_6U_2$. At the cubic level, quark mixing does not appear, but higher order couplings render quarks to mix [13].

(iv) *Fitting* $\sin^2\theta_W$: At the full unification scale, $\sin^2\theta_W$ is given by $\frac{3}{8}$ [13]. With a simple assumption that G-exotics are removed at M_{GE} and E-exotics are removed at M_{EE}, we fit M_{GE} and M_{EE} so that three gauge couplings fall in the observed values at the electroweak scale by package SOFTSUSY [24] which includes two loop effects. Certainly, there exist solutions for M_{GE} and M_{EE} around $M_{\rm GUT}$ where $M_{\rm GUT}$ is defined to be the converging point of α_2 and α_3. Above $M_{\rm GUT}$, at $g_{st}\Lambda_s$ a full unification of couplings is achieved. For example, a solution set is $M_{\rm GUT} = 2M_{16}, M_{EE} = 6M_{16}, M_{GE} = 0.1M_{16}, \Lambda_s \simeq 52.7M_{16}$ and $g_s = 0.8$ where $M_{16} = 10^{16}$ GeV [25].

In this letter, an $\mathcal{S}$MSSM spectrum is obtained in a $\mathbf{Z}_{12-I}$ orbifold construction, Eq. (3). The Yukawa coupling structure clarified how $\mathcal{S}$MSSM spectrum is possible. The $\sin^2\theta_W^0$ GUT value $\frac{3}{8}$ is phenomenologically allowed with exotics removed around the GUT scales. So, these exotics in general present in 'string flipped' SU(5) are not expected to be light.

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[1] A. J. Buras, J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. **B135**, 66 (1978).

[2] P. Langacker and M. Luo, Phys. Rev. **D44**, 817 (1991); C. Giunti, C. W. Kim, and U. Lee, Mod. Phys. Lett. **A6**, 1745 (1991); J. R. Ellis, S. Kelly, and D. V. Nanopoulos, Phys. Lett. **B260**, 131 (1991); U. Amaldi, W. de Boer, and H. Fürstenau, Phys. Lett. **B260**, 447 (1991).

[3] J. E. Kim, Phys. Lett. **B564**, 35 (2003).

[4] J. Ellis, L. Lopez, and D. V. Nanopoulos, Phys. Lett. **B247**, 257 (1990).

[5] A. N. Schellekens, Phys. Lett. **B237**, 363 (1990).

[6] Z. Kakushadze and S. H. H. Tye, Phys. Rev. Lett. **77**, 2612 (1996).

[7] K.-S. Choi and J. E. Kim, *Quarks and Leptons from Orbifolded Superstring* (Springer-Verlag, Heidelberg, Germany, 2006).

[8] I. Antoniadis, J. Ellis, J. S. Hagelin, and D. V. Nanopoulos, Phys. Lett. **B231**, 65 (1989); J. E. Kim, Phys. Lett. **B591**, 119 (2004); T. Kobayashi, S. Raby, and R.-J. Zhang, Phys. Lett. **B593**, 262 (2004).

[9] S. M. Barr, Phys. Lett. **B112**, 219 (1982); J.-P. Derendinger, J. E. Kim and D. V. Nanopoulos, Phys. Lett. **B139**, 170 (1984).

[10] Q. Shafi and Z. Tavartkiladze, hep-ph/0606188, and references therein.

[11] I. Antoniadis *et. al.*, in Ref. [8].

[12] J. Ellis, V. E. Meyers, and D. V. Nanopoulos, Phys. Rev. **D70**, 075015 (2004).

[13] J. E. Kim and B. Kyae, hep-th/0608086.

[14] L. E. Ibañez, J. E. Kim, H. P. Nilles, and F. Quevedo, Phys. Lett. **B191**, 282 (1987); L. E. Ibanez, J. Mas, H. P. Nilles and F. Quevedo, Nucl. Phys. **B301**, 157 (1988); J. A. Casas and C. Munoz, Phys. Lett. **B214**, 63 (1988).

[15] L. J. Dixon, J. A. Harvey, C. Vafa, and E. Witten, Nucl. Phys. **B261**, 678 (1985); Nucl. Phys. **B274**, 285 (1986); L. E. Ibanez, H. P. Nilles and F. Quevedo, Phys. Lett. **B187**, 25 (1987).

[16] V. G. Kac and D. H. Peterson, in *Anomalies, Geometry and Topology*, Proc. of Argonne-Chicago Conf. (1985), 276; T. I. Hollowood and R. G. Myhill, Int. J. Mod. Phys. **A3**, 899 (1988); K.-S. Choi, K. Hwang, and J. E. Kim, Nucl. Phys. **B662**, 476 (2003).

[17] Y. Katsuki, Y. Kawamura, T. Kobayashi, N. Ohtsubo, Y. Ono and K. Tanioka, Nucl. Phys. **B341**, 611 (1990).

[18] T. Kobayashi, S. Raby, and R.-J. Zhang, Nucl. Phys. **B704**, 3 (2005).

[19] W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, Nucl. Phys. B **712** (2005) 139.

[20] S. Hamidi and C. Vafa, Nucl. Phys. **B279**, 465 (1987); L. J. Dixon, D. Friedan, E. J. Martinec, and S. H. Shenker, Nucl. Phys. **B282**, 13 (1987); D. Friedan, E. J. Martinec and S. H. Shenker, Nucl. Phys. **B271**, 93 (1986); T. Kobayashi and N. Ohtsubo, Phys. Lett. **B245**, 441 (1990).

[21] In the remainder, the unspecified chirality means R.

[22] K. Hwang and J. E. Kim, Phys. Lett. **B540**, 289 (2002); B. Kyae and Q. Shafi, Phys. Lett. **B635**, 247 (2006).

[23] H. Georgi, Nucl. Phys. **B156**, 126 (1979).

[24] B.C. Allanach, Comput. Phys. Commun. 143, 305 (2002).

[25] We thank I.-W. Kim for providing these numbers.